Half-life and radioactive equilibrium

Physics | Modern Physics | Nuclear & particle physics

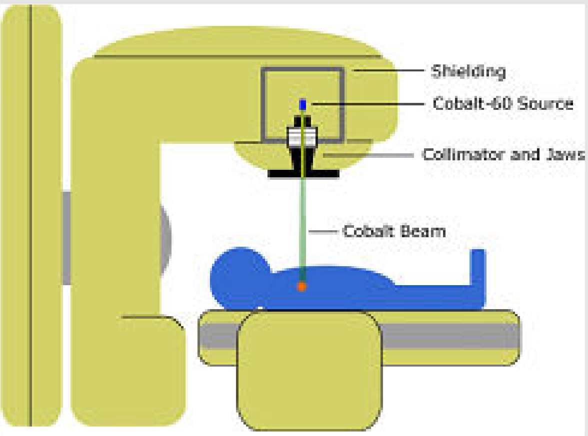
Difficulty level	Group size	Preparation time	Execution time
hard	2	45+ minutes	45+ minutes

This content can also be found online at:

http://localhost:1337/c/5f414d1b65140d000365ec59

General information

Application



Cancer treatment with Cobalt-60

Half-life is the time it takes for one-half of the atoms of a radioactive material to disintegrate. Certain radioactive materials are used in:

- Cancer treatment: Cobalt-60, Radium-226
- Biochemical tracer : Hydrogen-3
- Smoke detector: Americium-241

PHYWE
Robert-Bosch-Breite 10
37079 Göttingen
Tel.: 0551 604 - 0
Fax: 0551 604 - 107
info@phywe.de
www.phywe.de

Other information (1/2)

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Prior knowledge

Radioactive decay is the spontaneous breakdown of an unstable atomic nucleus resulting in the release of energy and matter from the nucleus. Three of the most common types of decay are alpha decay, beta decay, and gamma decay, all of which involve emitting one or more particles or photons.

Scientific principle

The half-life of a ^{137m}Ba daughter substance eluted (washed) out of a ^{137m}Cs isotope generator is measured directly and is also determined from the increase in activity after elution.

Other information (2/2)

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Learning objective

Understanding the properties behind the rate of radioactive decay in dependence upon time of reaction.

Tasks

1. Measure the activity of the isotope generator as a function of time immediately after elution.
2. Measure the activity of a freshly eluted solution of ^{137m}Ba as a function of time.

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Safety instructions

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For this experiment the general instructions for safe experimentation in science lessons apply.

For H- and P-phrases please consult the safety data sheet of the respective chemical.

Radioactive substances can be hazardous to your health! Always reduce the time spent handling radioactive substances to a minimum.

Do not eat or drink anything in the presence of radioactive substances.

Always wash your hands after contact with them!

Theory (1/8)

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The isotope generator contains 400 kBq of ^{137}Cs , which serves as the parent substance; its half-life is 30.25 years. ^{137}Cs decays into the barium isotope ^{137}Ba with the emission of β-radiation. This transition occurs, in part, directly (approximately 5%) to the stable ground state of ^{137}Ba and, in part (approximately 95%) via the meta stable state of ^{137m}Ba .

^{137m}Ba decays with a half-life of only 2.6 min under emission of γ-radiation ($E_\gamma = 662\,keV$) in the stable ground state of ^{137}Ba .

The decay chain is thus

$$N_1 \xrightarrow{\lambda_1} N_2 \xrightarrow{\lambda_2} N_3$$

where

N_1 = number of ^{137}Cs atoms
N_2 = number of ^{137m}Ba atoms
N_3 = number of ^{137}Ba atoms
λ_1 = disintegration constant of ^{137}Cs
λ_2 = disintegration constant of ^{137m}Ba .

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Theory (2/8)

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The rates of disintegration are:

$$\frac{dN_1}{dt} = -\lambda_1\, N_1$$

$$\frac{dN_2}{dt} = \lambda_1\, N_1 - \lambda_2\, N_2$$

Solving the system of two differential equations gives for $N_2(t)$, with the initial condition that $N_2(t=0)=0$:

$$N_2(t) = N_1(0)\,\frac{\lambda_1}{\lambda_2-\lambda_1}(e^{-\lambda_1\, t} - e^{-\lambda_2\, t})$$

Theory (3/8)

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The activity of the daughter substance is thus

$$A_2(t) = \lambda_2\, N_2 = A_1(0)\,\frac{\lambda_2}{\lambda_2-\lambda_1}(e^{-\lambda_1\, t} - e^{-\lambda_2\, t})$$

As the half-life of the parent substance $T_{1/2}(1)$ is much greater than that of the daughter substance $T_{1/2}(2)$ in this case, then $\lambda_1 \ll \lambda_2$. If we now neglect λ_1 in relation to λ_2 , we obtain

$$A_2(t) = A_1\,(1 - e^{-\lambda_2\, t})$$

with A_1 constant.

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Theory (4/8)

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When t is very long,

$$A_2(t) - A_1(t) \quad \text{and} \quad \lambda_2 N_2 = \lambda_1 N_1$$

i.e. the substances are in equilibrium (dynamic equilibrium, steady state).

If this equilibrium is disturbed by the removal of the daughter substance, the system will try to restore the equilibrium by forming more of it. For the increase in the daughter substance, we have:

$$A_2(t) = A_1(1 - e^{-\lambda_2 t})$$

with A_1 constant.

Theory (5/8)

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If the equilibrium activity

$$A_2(t = \infty) = A_1$$

is measured prior to elution, then λ_2 can be calculated from the measured values $A - A(t)$

$$A_1 - A_2(t) = A_1(e^{-\lambda_2 t})$$

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Theory (6/8)

The counting rate decreases to half of its original valu after a time period can be determined by

$$\dot{N} = \dot{N}_0 \, e^{-\lambda t}$$

and

$$\tfrac{1}{2}\,\dot{N} = \dot{N}_0 \, e^{-\lambda \tau} \quad .$$

Thus,

$$\lambda = \tfrac{\ln\ 2}{\tau} \quad .$$

$\dot{N}$: Actual rate of decay

$\dot{N}_0$: Counting rate at time

λ : Decay constant

t : Time

τ : Half-life

Theory (7/8)

By taking the logarithm of

$$\dot{N} = \dot{N}_0 \, e^{-\lambda t}$$

the following is obtained

$$\ln \dot{N} = -\lambda t \ \ln \ \dot{N}_0$$

Comparing the coefficients with the regression line $y = mtb$ shows that the slope of the regression line m corresponds to the negative decay constant:

$$m = -y$$

The decay constant λ is characteristic for the gradual decline in radioactivity: It is thus linked to the specific time period in which the substance quantity decays to half of its initial radioactivity (half-life τ).

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Theory (8/8)

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Formation of daughter substance subsequent to disturbance of the equilibrium by elution

Calculation of the half-life. At time t an activity is measured that can be calculated according to the following formula:

$$\dot{N}(t) = \dot{\overline{N}} + \dot{N}_0(1 - e^{-\lambda t}) \ .$$

$\dot{N}(t)$: Actual counting rate

$\dot{\overline{N}}$: Activity of the β-radiating ^{137}Cs (through the opining in the source) and the residual activity of the ^{137m}Ba, which results from incomplete elution.

$\dot{N}_0$: Equilibrium activity of the daughter substance, ^{137m}Ba

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Equipment

Position	Material	Item No.	Quantity
1	PHYWE Geiger-Müller Counter	13609-99	1
2	Geiger-Mueller counter tube, type B, BNC cable 50 cm	09005-00	1
3	Isotope generator Cs-137, 370 kBq	09047-60	1
4	Base plate for radioactivity	09200-00	1
5	Specimen tube with holder	09203-01	1
6	Plate holder on fixing magnet	09203-00	1
7	Source holder on fixing magnet	09202-00	1
8	Beaker, Borosilicate, low form, 250 ml	46054-00	2
9	Test tubes 100x12 mm,FIOLAX,100pc	36307-10	1
10	Rubber stopper,d=14.5/10.5mm, w/o	39253-00	1

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Setup and procedure

Setup

Experimental setup

Set up the experiment as shown in the figure.

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Procedure

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In accordance with the instructions the isotope generator is eluted into a glass beaker which should then be placed as far away from the counter tube as possible. Make a U-shaped cap from a strip of aluminium sheet and put it over the tube: it will absorb the electrons in the betadecay phase which would otherwise interfere with the experiment.

To measure the increase in activity it is advisable to read off the impulse rate every 30 seconds after elution (counting ratemeter time constant = 10 seconds). Activity and im-pulse count are sufficiently proportional at low impulse rates.

To measure the half-life of the ^{137m}Ba isotope, first elute the isotope generator in a test tube, then place it as far away from the rest of the equipment as possible. The counter tube (without the aluminium cap) can now be set up immediately in front of the bottom end of the test tube.

Evaluation (1/6)

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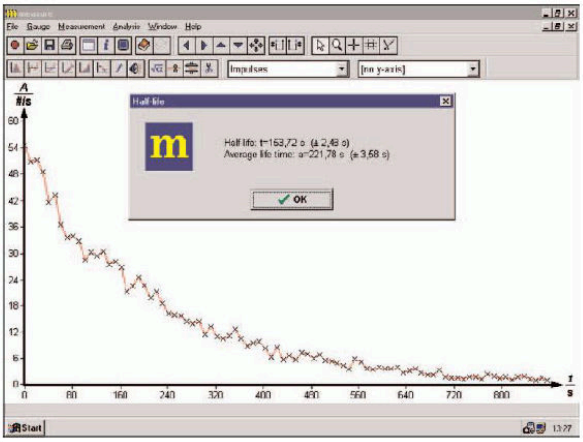

Calculation of the half-life of Ba-137m's decay, counting rate as a function of time

In the elution process, the ^{137m}Ba is washed out of the isotope generator. Figure shows the counting rate that decreases exponentially over time.

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Evaluation (2/6)

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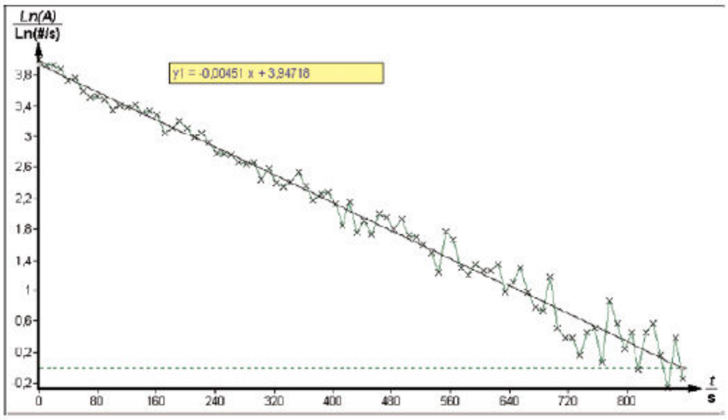


Logarithmic plot of the counting rate of Ba-137m’s decay with the regression line

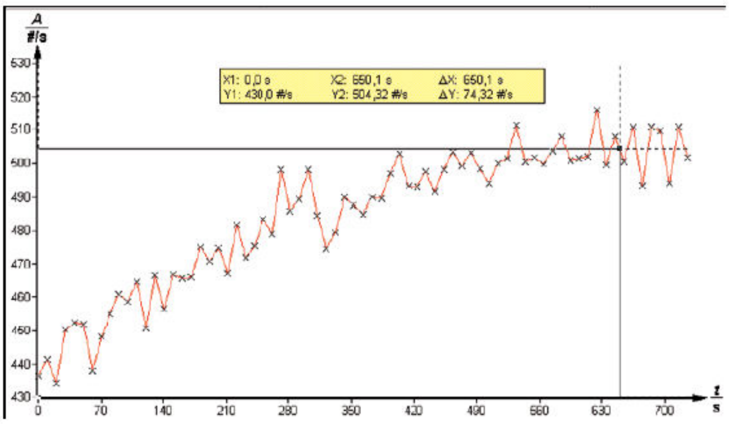
The logarithmic counting rate is plotted as a function of time, the straight line is obtained.

In this case, the counting rate decreases to half of its original value after a time period:

$\tau = 153.69\ s$.

Evaluation (3/6)

PHYWE



Plot of the counting rate of the ^{137m}Ba being formed as a function of time

By subtracting the equilibrium activity after approximately 650 s, which is comprised of $\dot{\overline{N}}$ and $\dot{N}_0$, from the respective actual counting rate, in this case 504.28 .

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Evaluation (4/6)

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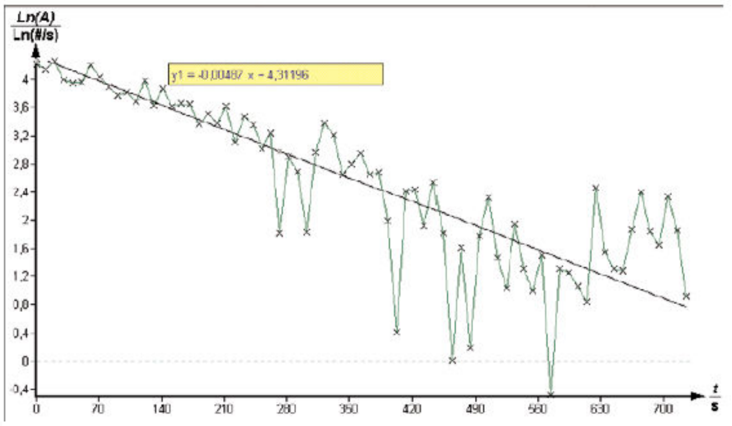

Logarithmic counting rateof the ^{137m}Ba being formed as a function of time, with the regression line

By calculating the natural logarithm of this variable's value, the Figure is obtained.

The half-life for this measurement, $\tau = 142.33\,\text{s}$ follows from the slope of the drawn regression line

$$\tau = -\frac{\ln\ 2}{m} = -\frac{\ln\ 2}{-0.00487} = 142.33\ s$$

Evaluation (5/6)

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Fill in the blanks:

Half-time of a nuclear decay process is the interval of ______ required for ______ of the unstable ______ of a radioactive sample to decay. The radioactive substance changes spontaneously into other nuclear species by emitting ______ and energy.

- atomic nuclei
- particles
- time
- one-half

Check

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Evaluation (6/6)

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For a given number of atoms, a radioactive sample with shorter half-lives decays

- [ ] more rapidly
- [ ] more slowly
- [ ] undergoing a greater number of radioactive decays at a given time

Check

The amount of radioactive decay is dependent on

- ○ Time
- ○ Temperature
- ○ Concentration of reactants

Check

Slide	Score / Total
Slide 22: Half time of decay process	0/4
Slide 23: Multiple tasks	0/3

Total Score 0/7

Show solutions Retry

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